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FREQUENCY COMPARISON OF A NASA EXPERIMENTAL HYDROGEN MASER WITH THE MEAN OF FIVE COMMERCIAL CESIUM STANDARDS

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ABSTRACT

Recent measurements of the long-term frequency stability of five commercial cesium beam frequency standards relative to the NASA NX-1 experimental hydrogen maser have yielded a fractional frequency difference between the two of $(6.77 \pm 1.66) \times 10^{-12}$. Since the frequency of the NX-1 maser was synthesized to the value for the hydrogen maser frequency given by Vessot et al. in 1966, this yields a new value for the hydrogen maser frequency of $1,420,405,751.7767 \pm .0024$ Hertz. This result is within the stated error limits of several other recent measurements of the hydrogen maser frequency.

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INTRODUCTION

Recent measurements of the long-term frequency stability of five commercial cesium beam frequency standards[1] relative to a NASA experimental hydrogen maser,[2] constructed at GSFC and designated NX-1, have made possible a new comparison of these two types of atomic frequency standards. The result of this comparison is a value for the hydrogen maser frequency that is 6.77 parts in 10^{12} lower than that given by Vessot et al.[3] in 1966.

The frequency comparison of the atomic standards is based on continuous phase difference measurements, sampled hourly, over the 123 days between May 7 and September 7, 1968. The 2952 hourly phase difference data thus obtained were punched on cards for computer analysis. Frequency data were obtained by taking the time derivative of successive five-data-point least-square-fits of the phase data, using Koenig's approximation formula.[4] The frequency data were then reduced to yield the mean fractional frequency difference between each cesium standard and the hydrogen maser. These figures were finally corrected so as to obtain the results in terms of the A-1 second.

DATA REDUCTION

To obtain the results shown in Table 1, the frequency data were least-square fitted (LSF) to a linear equation for frequency as a function of time. The fact that the total frequency change over the entire period of measurement due to the time dependent term was less than 1 σ indicates that the cesium beam standards exhibit no systematic linear frequency drift. However, in calculating the mean frequency difference, it was convenient to take the sum of the LSF frequency intercept at t=0 and one-half of the total frequency change due to the LSF time dependent term (these two components are given in Table 1). That this procedure correctly yields the mean of the frequency difference data can readily be proved. (See Appendix).

CORRECTIONS

The frequency output of the NX-1 hydrogen maser is synthesized to the value given by Vessot et al.[3] in 1966, corrected for differences in bulb shape, second order Doppler shift, and applied magnetic field. To achieve frequency

Table 1

Calculated Mean Fractional Frequency Difference Between

Cs Standards and Hydrogen Maser, from Phase Measurements (in parts per 10^{12})

Cs Units	(a) $\left(\dfrac{\Delta f}{f}\right)_{t=0}$	(b) $\delta\left(\dfrac{\Delta f}{f}\right)_{t=\frac{T}{2}}$	(c) $\left(\dfrac{\overline{\Delta f}}{f}\right)$	(d) σ
1	6.63	-.046	6.58	1.07
2	6.32	-.281	6.04	1.48
3	5.97	-.047	5.92	0.90
4	3.12	-.758	2.36	1.21
5	4.25	-.556	3.69	1.49
σ rms $= 1.25\times10^{-12}$ (RMS for all five Cs units)				

(a) $\left(\dfrac{\Delta f}{f}\right)_{t=0}$ is the intercept of an LSF linear equation for frequency as a function of time

(b) $\delta\left(\dfrac{\Delta f}{f}\right)_{t=\frac{T}{2}}$ is the average frequency change during the total period of measurement, T = 2952 hours.

(c) $\left(\dfrac{\overline{\Delta f}}{f}\right)$ is the mean fractional frequency difference between each cesium standard and the NASA hydrogen maser, NX-1.

(d) σ is the standard deviation of $\dfrac{\Delta f}{f}$ from its mean, $\dfrac{\overline{\Delta f}}{f}$.

outputs which are in terms of universal time, this synthesized NX-1 output and the output of each of the cesium standards are offset by an amount which is nominally -300×10^{-10}, the UT-2 offset for 1968, relative to the defined cesium resonance frequency based on the A-1 second.[5]

In the NX-1 this offset is maintained at exactly -300×10^{-10}. In the cesium standards, however, the amount of the offset was found to be slightly larger. Hence the measured fractional frequency difference between the cesium standards and the hydrogen maser must be corrected for the difference between the actual offset in the cesium standards and -300×10^{-10}.

The offset in the cesium standards is maintained by an internal electronic synthesizer and a magnetic field fine tuner. The first two columns of Table 2 show the contribution to the offset of each of these sources. The third column shows the effect on the offset of the magnetic field drift which was observed during the measurements. (The approximately linear behavior of this drift with time, shown in Figure 1, was used to compute its contribution to the offset.) The remaining columns of Table 2 show the total offset of each cesium standard and the appropriate correction to account for the difference between this offset and -300×10^{-10}.

Table 2

Magnetic Field and Synthesizer Offsets in
Cesium Standards and Consequent Corrections.

(The Off-Set Frequency of UT-2 for 1968 Relative to the A-1 Time
Scale is -300×10^{-10} Exactly.)

Cs Units	(a) Synthesizer Offset (parts per 10^{10})	(b) H Field Offset (parts per 10^{10})	(c) δH Field Offset (parts per 10^{12})	Total Offset (parts per 10^{10})	Correction to be applied to Fractional Frequency Difference (parts per 10^{12})
1	-301.8722	1.860	0.544	-300.0068	0.68
2	-301.8722	1.850	0.419	-300.0180	1.80
3	-301.8722	1.851	0.251	-300.0187	1.87
4	-301.8722	1.847	-0.034	-300.0260	2.60
5	-301.8722	1.849	----	-300.0230	2.30

(a) The synthesizer offset is furnished by the manufacturer.

(b) The magnetic field offset is calculated from the Breit-Rabi formula,
i.e. $\dfrac{\Delta f}{f} = 427H^2$ where $H = \dfrac{\nu}{350 \times 10^3}$, ν being the Zeeman frequency.

(c) The applied magnetic field in the transition region between the Ramsey cavities was found to exhibit a small upward drift, as shown in Figure 1.
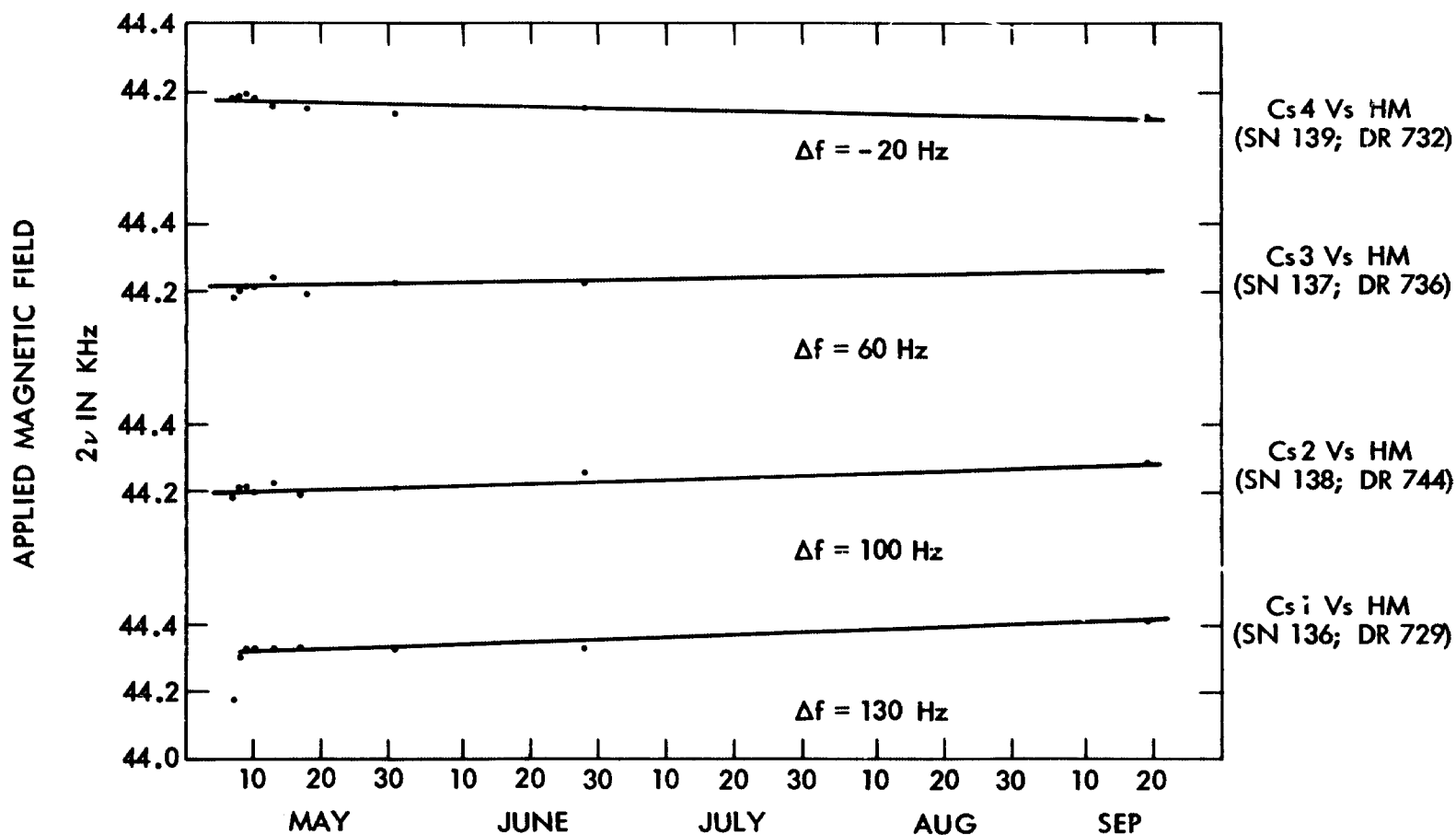
Figure 1 — Magnetic field measurements of cesium standards. (Δf is the total frequency change due to the magnetic field variation over the 130 days of field measurement; the serial number (SN) and current control potentiometer dial reading (DR) are given for each unit.)

After applying the corrections discussed above, the total fractional frequency difference between each cesium standard and the hydrogen maser, $\left(\overline{\dfrac{\Delta f}{f}}\right)_{Cs-HM}$, is as given in Table 3. The mean fractional frequency difference for the five units is 6.77×10^{-12}. The deviation, δ, of $\left(\overline{\dfrac{\Delta f}{f}}\right)_{Cs-HM}$ for each cesium standard from this mean is also given in Table 3, yielding a δ_{RMS} of $\pm 1.11 \times 10^{-12}$.

Table 3

Corrected Mean Fractional Frequency Difference between
Cesium Standards and Hydrogen Maser, in terms of the
Hydrogen Maser Frequency given by Vessot et al. (in parts per 10^{12})

Cs Units	$\overline{\dfrac{\Delta f}{f}}$	$\left(\dfrac{\Delta f}{f}\right)_c$	$\left(\overline{\dfrac{\Delta f}{f}}\right)_{Cs-HM}$	δ
1	6.58	0.68	7.26	0.49
2	6.04	1.80	7.84	1.07
3	5.92	1.87	7.79	1.02
4	2.36	2.60	4.96	-1.81
5	3.69	2.30	5.99	-0.78

$\left(\overline{\dfrac{\Delta f}{f}}\right)_{Cs-HM} = 6.77 \times 10^{-12}$ (Mean for all five Cs Units)

$\delta_{RMS} = 1.11 \times 10^{-12}$ (RMS for all five Cs Units)

ESTIMATED ERRORS

If the total error in the mean fractional frequency difference is taken to be the root-sum square of σ_{RMS}, from Table 1, and δ_{RMS}, from Table 3, a total error of 1.66×10^{-12} is obtained. This is consistent with the result obtained by combining the estimated absolute errors of the Hewlett-Packard cesium standards (1.33×10^{-12}) and the NX-1 hydrogen maser (1.0×10^{-12}), which yields a total of 1.64×10^{-12}. These individual errors are compared in Table 4 with those of other similar units.

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Table 4

Errors in Cesium and Hydrogen Maser Standards

Type of Standard	(a) σ 10^{-12}	Source	Comment
Cs (HP5060A-5061A)	1.33	H/P[6]	Based on factory test of 150 units relative to the company's cesium house standard, $3\sigma = 4 \times 10^{-12}$
Cs (NBS III)	1.1, 1.9	NBS[3,7]	Under normal conditions, $3\sigma = 5.6 \times 10^{-12}$; during the 1966 measurements with the Varian H-10 maser the NBS III received special care and its error was estimated at $1\sigma = 1.1 \times 10^{-12}$
Cs (NRC III)	.75	NRC[8]	Based on reevaluation of NRC Cs III, $2\sigma = 1.5 \times 10^{-12}$
H Maser (NASA NX-1)	1.0	NASA[9]	Based on repeated comparisons of NASA NX-1 and Varian H-10 masers
H Maser (NRC)	0.9	NRC[10]	Based on intercomparison of two hydrogen masers, HM-1 and HM-2
H Maser (H-10)	0.47	H/P[3]	Based on Varian H-10 #4

(a) It should be noted that the 1σ errors given in this table are not directly comparable, since the Hewlett-Packard figure represents the dispersion of a large number of production cesium standards, while the others are estimates, based on measurements of physical parameters which are known to affect the frequency, with the errors computed in a different way in each case.

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CONCLUSIONS

Our measured value for the fractional frequency difference between the commercial cesium standards and the NASA experimental maser is thus $(6.77 \pm 1.66) \times 10^{-12}$. Since the frequency of the NASA maser is synthesized to the 1966 measurement by Vessot et al., our value for the hydrogen maser frequency is therefore $f_{HM} = 1,420,405,751.7767 \pm .0024$ Hertz.

If this value is compared with two other measurements also made in 1968, those of A. G. Mungall et al.[10] and C. Menoud et al.[11] as shown in Table 5, it is evident that these three independent measurements agree within their stated errors.

It is also interesting to note that the three results shown in Table 5 fall within the error limits of the earlier results of Winkler[12] and of Peters et al.[13]. The discrepancy between these five measurements and the measurement of Vessot et al. indicates that further comparisons are desirable.

Table 5

Comparison of Recent Results of Hydrogen Maser Frequency
Measurements (since 1968) with the Value Given
by Vessot et al. (in 1966)

(Vessot et al. value = 1,420,405,751.7864 ± .0017 Hertz)

Source	Hydrogen Maser Frequency 1,420,405,751 +	Identifications
GSFC	.7767 ± .0024	H/P5060&5061 vs NASA NX-1
NRC	.7763 ±.0020	NRC HM-2 vs NRC CSIII[a]
LSRH	.7782 ± .0036	LSRH H-2 vs Ehanches CS-03[b]

(a) NRC = National Research Council, Ottawa Canada

(b) LSRH = Swiss Laboratory for Horological Research, Neuchatel, Switzerland

NOTES AND REFERENCES

1. The commercial cesium beam standards are manufactured by Hewlett-Packard Company. The first four units, which are all model 5061A, bear the serial numbers 136, 138, 137 and 139 respectively and were furnished to us for the determination of their long-term frequency stability by the Manned Flight Support Directorate. The fifth unit, model 5060A, serial number 152, is a laboratory standard.

2. H. E. Peters, E. H. Johnson, and T. E. McGunigal, "Atomic Hydrogen Standards for NASA Tracking Stations," Proc. 23rd Annual Frequency Control Symposium (May 1969).

3. R. Vessot, H. Peters, J. Vanier, R. Beehler, D. Halford, R. Harrach, D. Allan, D. Glaze, C. Snider, J. Barnes, L. Cutler, "An Intercomparison of Hydrogen and Cesium Frequency Standards," IEEE Trans. on Instr. & Meas. IM-15, 4, 165 (December 1966). Also, R. Beehler, D. Halford R. Harrach, D. Allan, D. Glaze, C. Snider, J. Barnes, R. Vessot, H. Peters, J. Vanier, Proc. IEEE 54, 2, 301 (February 1966).

4. D. M. Koenig, "Relative Phase Difference Analysis Program," Final Report, ASEE NASA Summer Faculty Fellowship Program, NASA Document X-520-68-179, pp. 77-88 (April 1968).

5. The 13th General Conference on Weights and Measures in 1967 adopted the definition of the second as that length of time which results in a value of exactly 9,192,631,770 Hertz for the transition frequency between the F=4, $M_F=0$, and F=3, $M_F=0$, hyperfine levels of cesium 133.

6. Private communication with L. N. Bodily. Based on the measurements of 150 units of Hewlett-Packard cesium standards during factory test, the 3σ fractional frequency distribution of all the units measured relative to the house standard is 4×10^{-12}. Through VLF phase comparison, the fractional frequency difference of the house standard relative to the frequency of the NBS III cesium standard is known to be within 1×10^{-12}.

7. Private communication with R. E. Beehler. Under normal care, the accuracy of the NBS III cesium standard is $3\sigma = 5.6 \times 10^{-12}$. When the Varian H-10 hydrogen maser was brought for comparison with NBS III in 1965 special care was given to the operation and error measurement of the cesium beam standard. The best estimate of the error of the NBS III during the 50 days of that comparison was $1\sigma = 1.1 \times 10^{-12}$.

8. A. G. Mungall, R. Bailey, H. Daams and D. Morris, "A Re-Evaluation of the NRC Long Cesium Beam Frequency Standard, "Metrologia 4, 4, 165 (October 1968).

9. Private communication with H. E. Peters. This value of σ is based on his repeated comparisons of the Varian H-10 and NASA NX-1 hydrogen masers. The second order Doppler shift, magnetic field and wall coating corrections were applied in all of his measurements.

10. A. G. Mungall, D. Morris, H. Daams, and R. Bailey, "Atomic Hydrogen Maser Development at the National Research Council of Canada," Metrologia 4, 3, 87 (July 1968). Also private communication with A. G. Mungall. The ±0.0020 Hertz uncertainty in the NRC measurement is based on the RMS uncertainties of $2\sigma = 1.5 \times 10^{-12}$ for NRC Cs III and $1\sigma = 1 \times 10^{-12}$ for hydrogen masers No. 1 and No. 2.

11. C. Menoud and J. Racine, "Stabilitat und Genauigkeit der Frequenz von Wasserstoff - Masern," Proc. 23rd Annual Frequency Control Symposium (May 1969).

12. Private communication with G. M. R. Winkler. His result for the hydrogen maser frequency, based on measurements carried out during 1965-66 between the U.S. Naval Research Laboratory's Varian H-10 Maser and the U.S. Naval Observatory's cesium maser clock, using a telephone link and averaged over a 20-day period, is 1420, 405,751.7765± .0020 Hertz.

13. H. E. Peters, J. Holloway, A. S. Bagley, L. S. Cutler, "Hydrogen Maser and Cesium Beam Tube Frequency Standards Comparison," Appl. Phys. Lett. 6, 2, 34 (Jan. 15, 1965). This comparison of a Varian hydrogen maser with Hewlett-Packard cesium standards yielded a value of 1420, 405,751.778 ± .016 Hertz for the hydrogen maser frequency. (The estimated error in this case was large enough to include both our result and the Vessot result.)

APPENDIX

PROOF OF FORMULA FOR MEAN FRACTIONAL FREQUENCY DIFFERENCE

As mentioned in the text, the frequency data obtained were least square fitted to an equation linear with time, i.e.

$$\frac{\Delta f}{f} = a_0 + a_1 t. \qquad \text{(A1)}$$

The mean fractional frequency difference, $\left(\overline{\frac{\Delta f}{f}}\right)$, was then computed using the formula

$$\left(\overline{\frac{\Delta f}{f}}\right) = a_0 + \frac{1}{2} a_1 T , \qquad \text{(A2)}$$

where $T = 2952$ hours, the total period of measurement. (The terms a_0 and $\frac{1}{2} a_1 T$ are given in columns 1 and 2 of Table 1 of the text; the resulting $\left(\overline{\frac{\Delta f}{f}}\right)$ is given in column 3.) The purpose of this appendix is to prove that equation (A2) is correct for the mean of $\left(\frac{\Delta f}{f}\right)$.

The linear least square fit of $\left\{(x_i, y_i)\right\}_{i=1}^{n}$ is given by

$$y = a_0 + a_1 x \qquad \text{(A3)}$$

where

$$a_0 = \frac{\begin{vmatrix} \Sigma y_i & \Sigma x_i \\ \Sigma x_i y_i & \Sigma x_i^2 \end{vmatrix}}{\begin{vmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{vmatrix}}$$

$$= \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2} \qquad \text{(A4)}$$

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and

$$a_1 = \cfrac{\begin{vmatrix} n & \Sigma y_i \\ \Sigma x_i & \Sigma x_i y_i \end{vmatrix}}{\begin{vmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{vmatrix}}$$

$$= \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - (\Sigma x_i)^2}. \tag{A5}$$

Theorem. Let $y = a_0 + a_1 x$ be a linear LSF of $\{(x_i, y_i)\}_{i=1}^n$. Then

$$\frac{1}{n} \sum_{i=1}^n y_i = a_0 + a_1 \left(\frac{1}{n} \sum_{i=1}^n x_i \right) \tag{A6}$$

Proof:

$$a_0 + a_1 \left(\frac{1}{n} \Sigma x_i \right) = \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i}{n \Sigma x_i^2 - \left(\Sigma x_i \right)^2} + \frac{n \Sigma x_i y_i - \Sigma x_i \Sigma y_i}{n \Sigma x_i^2 - \left(\Sigma x_i \right)^2} \left(\frac{\Sigma x_i}{n} \right)$$

$$= \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i + \Sigma x_i \Sigma x_i y_i - \frac{1}{n} \left(\Sigma x_i \right)^2 \Sigma y_i}{n \Sigma x_i^2 - \left(\Sigma x_i \right)^2}$$

$$= \frac{\left[n \Sigma x_i^2 - \left(\Sigma x_i \right)^2 \right] \left(\frac{1}{n} \Sigma y_i \right)}{n \Sigma x_i^2 - \left(\Sigma x_i \right)^2}$$

$$= \frac{1}{n} \sum_{i=1}^n y_i \tag{A7}$$

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Now consider the case of equally spaced abscissa values, i.e. where

$$x_i = x_1 + (i - 1)h,$$
(A8)

for $1 \leq i \leq n$. This permits the following simplification of the average of the x_i's:

$$\frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1}{n} \sum_{i=1}^{n} [x_i + (i - 1)h]$$

$$= \frac{1}{n} \left[nx_1 + \frac{n(n - 1)h}{2} \right]$$

$$= x_1 + \frac{(n - 1)h}{2}$$

$$= \frac{1}{2} \left\{ x_1 + [x_1 + (n - 1)h] \right\}$$

$$= \frac{x_1 + x_n}{2} \ .$$

Substituting this result into (A6) we have

$$\frac{1}{n} \sum_{i=1}^{n} y_i = a_0 + a_1 \left(\frac{x_1 + x_n}{2} \right),$$
(A9)

i.e., for equally spaced abscissa values, the value of the LSF at the midpoint of the interval $[x_1, x_n]$ is the average of the y_i, $1 \leq i \leq n$.

In our case $x = t$ and $y = \frac{\Delta f}{f}$; hence, taking $x_1 = 0$ and $x_n = T$ and using the definition

$$\left(\overline{\frac{\Delta f}{f}} \right) = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{\Delta f}{f} \right)_i,$$
(A10)

equation (A9) reduces to equation (A2), Q.E.D.